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MONETARY AND FISCAL POLICY IN AN OPTIMISING MODEL WITH CAPITAL ACCUMULATION AND FINITE LIVES*

Giancarlo Marini and Frederick van der Ploeg

The theoretical debate on the long-run effectiveness of changes in monetary growth seems to have reached a stalemate in the last two decades. The original result obtained by Tobin (1965), in a classical macroeconomic model is, for example, unchanged in Lucas's (1975) new classical equilibrium business cycle model. Money is not superneutral, for the long-run capital labour ratio is positively related to the anticipated rate of growth of the nominal supply of money.[1]

However, the existence of any such a relationship has been denied by a strand of literature focusing on optimising models with microeconomic foundations. The long-run superneutrality of money was originally proved by Sidrauski (1967*b*) in a model where agents have an inelastic supply of labour, derive utility from consumption and real balances, and form expectations adaptively. This result has been replicated in a perfect-foresight framework, although non-neutralities can occur during the transient path towards long-run equilibrium (Fischer, 1979*b*; Asako, 1983). Long-run non-neutralities can, of course, still be obtained by introducing money in the production function (e.g. Dornbusch and Frenkel, 1973; Fischer, 1974), by including leisure into the utility function in a non-separable fashion (e.g. Brock, 1974), or by allowing for population growth (e.g. Weil, 1986). The procedure of entering real money balances into the direct utility function has often been criticised. However, it has gained new appeal in the light of the 'amenity value' argument advanced by McCallum (1986) and, especially, the taxonomic analysis of Feenstra (1986). The latter study formally demonstrates its equivalence with explicitly considering liquidity costs in the households' budget constraint. This strengthens the superneutrality result obtained by Sidrauski (1967*b*) and others in optimising models.

The main aim of this paper is to analyse the robustness of the result that money is superneutral in models where real money balances enter the direct utility function but not the production function (thus excluding the non-neutralities discussed in Fischer, 1974; Dornbusch and Frenkel, 1973) and where utility is not separable in leisure (thus excluding the non-neutralities

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[1] The same result is discussed in Sidrauski (1967*a*), who uses adaptive expectations, and Fischer (1979*a*) and Begg (1980), who use rational expectations.

discussed in Brock, 1974). In particular, the objective is to consider models where the planning horizon of private sector agents is finite and the horizon of the government is infinite. One approach would be to introduce money into an overlapping-generations framework (e.g. Diamond, 1965),[2] but we prefer to base our analysis on uncertain lifetimes along the lines suggested by Yaari (1965) and Blanchard (1985). The main advantage of the Blanchard Yaari framework is that it is much easier to obtain the intertemporal optimising model used by Sidrauski (1967*b*) and others as a special case and therefore it is easier to compare results.

The scheme of the paper is as follows. Section I develops a model of consumers with uncertain lifetimes and of firms and derives the aggregate equilibrium relationships and budget constraints for the economy. The asset menu consists of money, bonds and equity. Section II assumes that the residual mode of government finance is lump-sum taxation and that real bonds are constant. This case corresponds most closely to the money-capital economies discussed by Sidrauski (1967*b*) and others, although we show that finite horizons ensure that an increase in monetary growth increases capital and output and reduces real money balances. An increase in government spending leads to crowding out of private sector consumption, a fall in capital and output, and a fall in real money balances, even though with infinite lives fiscal policy is neutral. Section III considers money-financed fiscal policy. Section IV assumes that the residual mode of government finance is bonds and that lump-sum taxes adjust (only in the short run) to stabilise the government debt. In that case, monetary policy is superneutral even with finite lives. However, if at any point of time preferences are non-separable in consumption of goods and real money balances *and* lives are finite, non-neutralities occur. Section V concludes the paper with a summary of the results.

I. AN OPTIMISING MODEL WITH UNCERTAIN LIFETIMES

The demand side of the economy consists of identical agents with constant life expectancy. In addition, there is no bequest motive, as in the analysis of Blanchard (1985). There is no population growth, so that the birth and death rates are the same.[3] The supply of labour at time t of an agent born at time $s \leqslant t$, $l(s,t)$, is inelastic, say $l(s,t) = 1$. Assuming a logarithmic utility function, the problem faced by the representative agent is the following:

$$\underset{x(s,v)}{\mathrm{Max}} \int_t^{\infty} \log\{\Omega[c(s,v), m(s,v)]\} \exp[(\alpha+\beta)(t-v)]\, dv, \tag{1}$$

subject to the individual's flow budget constraint

$$\frac{da(s,t)}{dt} = [r(t)+\beta]\, a(s,t) + w(t)\, l(s,t) - z(s,t) - x(s,t) \tag{2}$$

[2] Aiyagari and Gertler (1985) use such a framework to discuss the validity of various monetarist propositions under alternative budgetary policies.

[3] An alternative is to have population growth (Weil, 1986; Buiter, 1986), because what matters is a non-zero birth rate so that taxes can be passed on to future, yet unborn, generations.

and the condition precluding private agents' Ponzi games,

$$\lim_{v \to \infty} \exp\left\{-\int_t^v [r(\mu)+\beta]\, d\mu\right\} a(v,t) = 0, \tag{3}$$

where α denotes the rate of time preference, β denotes the constant probability of death, $w(t)$ denotes the real wage rate at time t, $r(t)$ denotes the real interest rate at time t, and $c(s,t)$, $m(s,t)$, $a(s,t)$, $z(s,t)$ and $x(s,t)$ denote the consumption of goods, real money balances, total real non-human wealth, lump-sum taxation levied and total consumption at time t of an individual born at time s, respectively. Note that the individual receives, for every period of his life, a premium $\beta a(s,t)$, which is actuarially fair, and that the individual's net wealth at the time of death goes to the insurance company. Since the probability of death is β, the subjective rate of time preference is effectively increased by this amount (Yaari, 1965; Blanchard, 1985). Total consumption consists of physical consumption plus interest foregone on money holdings, that is

$$x(s,t) \equiv c(s,t) + [r(t)+p(t)]\, m(s,t) = q(t)\, u(s,t), \tag{4}$$

where $p(t)$ and $q(t)$ denote the inflation rate and the ideal cost-of-living index of the basket of physical goods and real money balances at time t, respectively, and $u(s,t) = \Omega(\cdot)$ denotes the instantaneous utility at time t of an individual born at time s, respectively. We assume that the private agent's decision problem can be split into two stages. In the first stage the individual decides on total consumption and in the second stage he decides how to allocate it between consumption of physical goods and consumption of real money balances.

The first stage yields that the marginal rate of substitution between total consumption and non-human wealth equals the ideal price index of the basket of total consumption

$$\frac{1}{u(s,t)} = \lambda(s,t)\, q(t). \tag{5}$$

It also yields the 'tilt' of the consumption function,

$$\frac{d\lambda(s,t)}{dt} = [\alpha - r(t)]\, \lambda(s,t), \tag{6}$$

where the following transversality condition must be satisfied,

$$\lim_{v \to \infty} \lambda(s,v)\, a(s,v) \exp[(\alpha+\beta)(t-v)] = 0. \tag{7}$$

Total wealth must equal the present discounted value of future expenditure on total consumption:

$$a(s,t) + h(s,t) = \int_t^{\infty} q(v)\, u(s,v) \exp\left\{-\int_t^v [r(\mu)+\beta]\, d\mu\right\} dv, \tag{8}$$

where human capital is defined as the present discounted value of after-tax wage income,

$$h(s,t) \equiv \int_t^{\infty} [w(v)\, l(s,v) - z(s,v)] \exp\left\{-\int_t^v [r(\mu)+\beta]\, d\mu\right\} dv. \tag{9}$$

It follows from (5) and (8) that

$$x(s,t) = [\lambda(s,t)]^{-1} = (\alpha+\beta)[a(s,t)+h(s,t)], \tag{10}$$

so that total consumption is linear in wealth and aggregation across agents born at the same instant is feasible. The linear consumption function is a result of the assumption that the utility function, (1), is unit-elastic over time. Blanchard (1985) discusses the effects of the general iso-elastic utility function for non-monetary economies. After aggregation over all the cohorts of consumers and using the fact that the non-human wealth of newly born agents is zero, $a(t,t) = 0$, one obtains the dynamics of human and non-human wealth:

$$\dot{H} = (r+\beta)H - w + Z, \tag{11}$$

$$\dot{A} = rA + w - (\alpha+\beta)(A+H) - Z, \tag{12}$$

where $H(t) \equiv \int_{-\infty}^{t} h(s,t)\beta\exp[\beta(s-t)]\,ds$ denotes aggregate human wealth, $A(t) \equiv \int_{-\infty}^{t} a(s,t)\beta\exp[\beta(s-t)]\,ds$ denotes aggregate non-human wealth and $Z(t) \equiv \int_{-\infty}^{t} z(s,t)\beta\exp[\beta(s-t)]\,ds$ denotes aggregate taxes. An alternative representation of (11) and (12) is:

$$\dot{U} = (r-\alpha)U - \beta(\alpha+\beta)A, \tag{11'}$$

$$\dot{A} = rA + w - Z - U, \tag{12'}$$

where $U(t) \equiv q(t)\int_{-\infty}^{t} u(s,t)\beta\exp[\beta(s-t)]\,ds$ denotes the aggregate value of total consumption. Note that U (or H) is a forward-looking variable.

The second stage of the optimisation problem assumes a homothetic sub-utility function, $u(s,t) = \Omega[c(s,t), m(s,t)]$, which is maximised with respect to consumption and real money balances subject to the instantaneous private agent's budget constraint, (4). The sub-utility function can, without loss of generality, be assumed to be homogeneous of degree one. The individual ensures that the marginal rate of substitution between consumption of physical goods and real money balances equals the opportunity cost of holding real money balances (i.e. the nominal interest rate), $\Omega_m/\Omega_c = r+p$, so that $c(s,t) = \Gamma[r(t)+p(t)]\,m(s,t)$ where $\Gamma' > 0$. Upon substitution into (4) and aggregation, one obtains

$$M = \Psi(r+p)\,U, \quad \Psi(r+p) \equiv [r+p+\Gamma(r+p)]^{-1}, \tag{13}$$

where $\Psi' = -(1+\Gamma')\Psi^2 < 0$, and

$$C = \Phi(r+p)\,U, \quad \Phi(r+p) \equiv \Gamma(r+p)\Psi(r+p), \tag{14}$$

where $\Phi' = [(r+p)\Gamma' - \Gamma]\Psi^2 \gtrless 0$. For example, a CES sub-utility function,

$$\Omega(\cdot) = [\gamma c(s,t)^{\eta} + (1-\gamma)m(s,t)^{\eta}]^{1/\eta}, \quad (\eta \neq 1, 0 < \gamma < 1),$$

yields

$$\Gamma(\cdot) = [\gamma(r+p)/(1-\gamma)]^{\sigma}, \; \Psi(\cdot) = (1-\gamma)[(1-\gamma)(r+p)+\gamma(r+p)^{\sigma}]^{-1}$$

and

$$\Phi(\cdot) = \{1+[(1-\gamma)/\gamma]^{\sigma}(r+p)^{1-\sigma}\}^{-1},$$

where $\sigma \equiv (1-\eta)^{-1}$ denotes the elasticity of substitution between physical goods and real money balances. Homotheticity ensures that physical goods and real money balances are normal goods. When the sub-utility function is weakly separable, $V' = (r+p)V''$ and therefore $\Phi' = 0$. This is the case for a Cobb Douglas sub-utility function ($\sigma = 1$), since then $\Phi(\cdot) = \gamma$. In general, consumption of physical goods is a decreasing (increasing) function of the nominal interest rate when the income (substitution) effect dominates. For a CES sub-utility function, this happens when $\sigma < 1$ ($\sigma > 1$).

The production side of the economy is modelled in a standard way. The value of the firm, V, follows from the condition for risk-neutral arbitrage between equity and other assets, $rV = \dot{V} + Y - wL - I$, where Y, L and I denote aggregate production of physical goods, employment and gross investment of the firm, respectively. Under perfect foresight the value of the firm equals the present discounted value of future profits:

$$V(t) = \int_t^{\infty} [Y(v) - w(v)\,L(v) - I(v)]\exp\left[-\int_t^{v} r(\mu)\,d\mu\right]dv. \tag{15}$$

The firm chooses L and I to maximise the value of the firm subject to

$$\dot{K} = I - \delta K,\; K(0) = K_0 \tag{16}$$

and a concave constant-returns-to-scale production function, $Y = F(K, L)$, where K denotes the capital stock and $\delta \geqslant 0$ denotes the rate of depreciation. This yields $F_L(K, L) = w$ and $F_K(K, L) = r + \delta$, where $r + \delta$ is the user cost of capital.[4]

Equilibrium in, respectively, the labour, goods and money markets can be characterised by: $L = 1$,

$$Y = C + I + G, \tag{17}$$

$$\dot{M} = (\theta - p)\,M, \tag{18}$$

where G denotes real government spending and θ denotes the rate of growth in the nominal supply of money. The asset menu consists of real money balances, M, real government indexed bonds, B, and real equity, V, so that the real value of the private sector's non-human wealth is given by $A = V + M + B$. Solvency of the government's budget implies that the current government debt has to be paid off by the present discounted value of the excess of future lump-sum tax and seignorage revenues over public spending:

$$B_0 = \int_0^{\infty} [Z(t) + \theta(t)\,M(t) - G(t)]\exp\left[-\int_0^{t} r(v)\,dv\right]dt. \tag{19}$$

The government has four policy instruments, viz. G, Z, θ and B, of which three can be chosen freely and the fourth follows residually from the government's budget constraint.

[4] There are no adjustment costs in investment, that is Tobin's 'Q' is unity, and therefore $V = K$. Note that wage plus dividend income can be written as $wL + (Y - wL - I) = Y - \delta K - \dot{K} = rK + wL - \dot{K}$, which has been used in the individual's budget constraint, (2).

II. FINANCE BY LUMP-SUM TAXATION

II.1. *Monetary Policy*

In this section we analyse the effects of alternative monetary growth rates and levels of public spending under a tax-finance regime. For simplicity, we assume that the exogenous stock of outstanding bonds is zero, $B = 0$. The government's budget constraint, (19) and (18) give the residual amount of lump-sum taxation as the excess of government spending over seignorage revenues, $Z = G - \theta M$. Since government spending is assumed to be exogenous, the government chooses monetary growth and lump-sum taxes adjust to balance the government budget. The dynamics of this economy can be described by the following three differential equations:

$$\dot{K} = F(K, 1) - \Phi[\Psi^{-1}(M/U)]U - \delta K - G, \quad K(0) = K_0, \tag{20}$$

$$\dot{U} = [F_K(K, 1) - \delta - \alpha]U - \beta(\alpha+\beta)(K+M), \quad U(0) = \text{free}, \tag{21}$$

$$\dot{M} = M[F_K(K, 1) - \delta + \theta - \Psi^{-1}(M/U)], \quad M(0) = \text{free}, \tag{22}$$

where K is a predetermined variable and U and M are jump variables.

In steady-state inflation is given by monetary growth, $p = \theta$, and the other multipliers are:

$$\left.\frac{dK(\theta)}{d\theta}\right|^{TF} = \beta(\alpha+\beta)(A\Gamma\Psi' + K\Gamma'\Psi)/\Delta, \tag{23}$$

$$\left.\frac{dM(\theta)}{d\theta}\right|^{TF} = [-\Gamma' M^2 F_{KK} + \beta(\alpha+\beta)(rA\Psi' + \Gamma' Mr\Psi)]/\Delta, \tag{24}$$

where $\Delta \equiv \Psi\beta(\alpha+\beta)(K/M)(r - M\Gamma' F_{KK}) + \Gamma[F_{KK}M - \beta(\alpha+\beta)(\Psi + \Psi' F_{KK} A)]$.

Consider the special case of a zero birth and death rate, $\beta = 0$. In that case the real interest rate must equal the rate of time preference, $r = \alpha$, and therefore the capital stock is independent of the monetary growth rate, $F_K(K, 1) = \alpha + \delta$ and $[dK(\infty)/d\theta]^{TF} = 0$. Furthermore, the consumption of physical goods is unaffected. This is, of course, the well-known Sidrauski (1967*b*) result. An increase in monetary growth increases the nominal interest rate one-for-one and therefore reduces holdings of real money balances,

$$[dM(\infty)/d\theta]^{TF} = -\Gamma' M/\Gamma = -\sigma M/(r+\theta) < 0.$$

It can easily be shown that real seignorage revenues, θM, increase, so that lump-sum taxes fall and human wealth increases. In fact, the fall in non-human wealth, caused by the reduction in real money balances, is exactly off-set by the increase in human wealth, so that total wealth and the consumption of physical goods are unaffected.

When the birth and death rate is zero, i.e. lives are infinite ($\beta = 0$), *and* the sub-utility function is weakly separable in c and m ($\Phi' = 0$, e.g. when the sub-utility function is Cobb Douglas), the real part of the dynamic system separates out from the monetary part as $\Phi[\Psi^{-1}(M/U)] = \gamma$. Clearly, θ does not affect (20)–(21) and therefore monetary growth does not affect the transient dynamics of K, Y, C, V or U either. However, when the sub-utility function is not weakly separable, monetary growth affects the marginal propensity to consume goods out of total wealth and thus affects the transient dynamics (and eigenvalues), but it does not affect the steady-state value of capital (cf. Fischer, 1979*b*; Asako, 1983).

Now consider the general case of finite lives, $\beta > 0$. Equation (23) shows that changes in monetary growth affect the steady-state value of the capital stock. Even a weakly separable sub-utility function generates this non-neutrality result. In order to sign the steady-state multipliers unambiguously, the remainder of this section assumes a weakly separable (Cobb Douglas) sub-utility function ($\sigma = 1$):

$$\left.\frac{dK(\infty)}{d\theta}\right|^{TF} = -\beta(\alpha+\beta)\,\gamma M(r+\theta)^{-1}\Delta^{-1} \gtrless 0, \tag{23'}$$

$$\left.\frac{dM(\infty)}{d\theta}\right|^{TF} = \{-(r+\theta)\,M\Delta + \beta(\alpha+\beta)\,[\gamma M^2 F_{KK} - (1-\gamma)\,rM]\}/[(r+\theta)^2\,\Delta] \lesseqgtr 0, \tag{24'}$$

where the saddlepoint property requires that $\Delta < 0$ (e.g. Buiter, 1984). Hence, with finite lives and positive birth and death rates an increase in monetary growth leads in the long run to an equal increase in inflation, a fall in the real interest rate, an increase in capital, output and consumption of physical goods, and either a fall or, for a large probability of death, an increase in real money balances. This effect is very similar to the conventional Mundell Tobin effect, yet it is derived from a general equilibrium model with micro foundations. This non-neutrality arises, because with finite lives a wedge is driven between the discount rate used to calculate government debt, r, and the one used to calculate human wealth, $r+\beta$, and consequently the long-run real interest rate is endogeneous and need no longer equal the rate of time preference.

The long-run effect on real seignorage revenue is

$$\left.\frac{d[\theta M(\infty)]}{d\theta}\right|^{TF} = M + \theta\left.\frac{dM(\infty)}{d\theta}\right|^{TF} = \{r(r+\theta)\,M\Delta + \beta(\alpha+\beta)\,[\gamma M^2 F_{KK} - r(1-\gamma)\,M]\}/[(r+\theta)^2\,\Delta] > 0, \tag{25}$$

so that an increase in monetary growth raises seignorage revenues (despite a possible fall in real money balances) and therefore reduces lump-sum taxes. Human wealth increases, because wage income increases, lump-sum taxes fall and the real interest rate falls. The increase in human wealth more than off-sets any possible fall in real money balances and non-human wealth, so that total wealth and therefore consumption rises. Because the real rate of interest exceeds the rate of time preference, individual consumers save relatively more in the early part of their life and consume relatively more later on. The effects of monetary growth on social welfare are ambiguous. However, if the probability of death is high enough, an increase in monetary growth can increase real money balances and can therefore increase social welfare as the consumption of physical goods always increases.

II.2. *Fiscal Policy*

The steady-state effects of tax-financed fiscal policy on capital, real money balances and consumption are:

$$\left.\frac{dK(\infty)}{dG}\right|^{TF} = \beta(\alpha+\beta)\,(K/M)\,\Psi/\Delta \leqslant 0, \tag{26}$$

$$\left.\frac{dM(\infty)}{dG}\right|^{TF} = [\beta(\alpha+\beta)\,(\Psi+\Psi' F_{KK}A)-F_{KK}M]/\Delta < 0, \tag{27}$$

$$\left.\frac{dC(\infty)}{dG}\right|^{TF} = -\left[\Delta-\beta(\alpha+\beta)\left(\frac{K}{M}\right)\Psi r\right]\Big/\Delta \leqslant -1. \tag{28}$$

With infinite lives, an increase in public spending is completely crowded out by a fall in private consumption of physical goods ($[dC(\infty)/dG]^{TF} = -1$) induced by a fall in human capital, holdings of real money balances and therefore private wealth. Hence, there is no effect on the real interest rate, capital or output. However, with finite lives, real money balances, non-human wealth, human wealth and private consumption of physical goods fall by a greater amount and therefore there is a rise in the real interest rate, crowding out of private investment and a fall in output. Note that lump-sum taxes also have to finance the drop in seignorage revenues, so that they have to increase by more than the increase in public spending. Human wealth falls for three reasons: wage income falls as the fall in capital reduces the marginal productivity of labour; lump-sum taxes increase; and the real interest rate rises. An increase in public spending unambiguously reduces social welfare, because consumption of both physical goods and of real money balances declines. Obviously, this is due to the assumption that public goods are of the 'hole-in-the-ground' variety or do not enter the utility function of private agents (in a non-separable fashion).[5]

III. MONEY-FINANCED FISCAL POLICY

Since the seminal work of Blinder and Solow (1973) and of Tobin and Buiter (1976), the relative effectiveness of fiscal policy financed by monetary creation or by new bond issues has been an active policy debate. In this section we examine the first case of money-financed changes in government spending in terms of our optimising model with capital accumulation, flexible prices and finite lives. Section IV.2 discusses bond-financed fiscal policy.

Both the level of exogeneous lump-sum taxes and the stock of outstanding

[5] A detailed analysis of the welfare effects of government spending in a disequilibrium framework is provided by Rankin (1987).

government debt are normalised to zero. The steady-state multipliers are given by:

$$\left.\frac{dK(\infty)}{dG}\right|^{MF} = -\beta(\alpha+\beta)\, w/[U \det(J')] \geqslant 0, \tag{29}$$

$$\left.\frac{dU(\infty)}{dG}\right|^{MF} = -[rU+\beta(\alpha+\beta)\, M]\, F_{KK}/\det(J') < 0, \tag{30}$$

$$\left.\frac{dM(\infty)}{dG}\right|^{MF} = [-U^2F_{KK}+\beta(\alpha+\beta)\,(w-MAF_{KK})]/[U\det(J')] < 0, \tag{31}$$

where $\det(J') = \gamma rUF_{KK}+\beta(\alpha+\beta)\,(\gamma MF_{KK}-rw/U) < 0$. Classical neutrality propositions only hold when the economy is populated by infinitely-lived agents. That is, when $\beta = 0$, the steady-state level of capital is unaffected by money-financed fiscal policy. However, with finite horizons, there is less than 100% crowding out of private consumption as a money-financed fiscal expansion increases capital in the long run. The economic intuition is as follows.

Real seignorage revenues, M, have to rise by the same amount as government spending, hence monetary growth and inflation have to increase proportionally by more than the fall in real money balances. This increase in monetary growth leads to the type of Mundell–Tobin effects discussed in Section II.1, so that the real interest rate falls and capital increases. Hence, the increase in inflation helps to choke off the increase in the demand for goods. The increase in capital raises the wage and the fall in the real interest rate raises the annuity value of wage income, so that human wealth increases. Non-human wealth decreases,

$$\left.\frac{dA(\infty)}{dG}\right|^{MF} = -[U^2+\beta(\alpha+\beta)\, MA]\, F_{KK}/[U\det(J')] < 0, \tag{32}$$

since the fall in real money balances dominates the increase in equity. In fact, the fall in real money balances is large enough to ensure that total private wealth, $A+H$, and therefore total consumption, U, declines. Consumption of physical goods also falls, but with finite lives it does not completely crowd out the increase in government spending as the increase in supply must be matched by an increase in demand for goods:

$$-1 \leqslant \left.\frac{dC(\infty)}{dG}\right|^{MF} = r\left.\frac{dK(\infty)}{dG}\right|^{MF} - 1 = -\gamma[rU+\beta(\alpha+\beta)\, M]\, F_{KK}/\det(J') < 0. \tag{33}$$

Holdings of real money balances fall, because total consumption and total private wealth fall and because the opportunity cost of holding real money balances increases:

$$\left.\frac{d[r(\infty)+\theta(\infty)]}{dG}\right|^{MF} = \{(U/M)\,[(\gamma r+\theta)/(1-\gamma)]\, F_{KK} - \beta(\alpha+\beta)\,[(w/M)-KF_{KK}]/M\}/\det(J') > 0, \tag{34}$$

despite the fall in the real interest rate.

It follows that a money-financed fiscal expansion does not seem to be desirable in spite of the fact that the steady-state level of the capital stock increases. The reason is that capital accumulation is obtained at the expense of higher inflation and a higher nominal interest rate. The associated fall in real money balances depresses total wealth and consumption and thus unambiguously reduces social welfare.

IV. FINANCE BY ISSUING BONDS

Bond finance is unstable unless rules for spending, taxation or monetary growth that prevent the escalation of government debt are specified. Without such a rule the solvency (or no Ponzi games) condition for the government will not be satisfied. Since we are interested in monetary and fiscal policy, we adopt a rule for lump-sum taxes of the form (Buiter, 1986):

$$Z = \xi_0 - \xi_1 \dot{B} = -[\xi_0 - \xi_1(rB + G - \theta M)]/(\xi_1 - 1) \quad (\xi_1 > 1), \tag{35}$$

so that taxes are raised when there is a government surplus. Lump-sum taxes are constant, ξ_0, in the long run and the choice of ξ_1 does not affect the steady-state effects of monetary and fiscal policy. Note that, for $\theta = 0$, a long-run increase in taxation, ξ_0, must be preceded by a short-run cut in taxation. Over time bonds and therefore taxes increase until the interest payments on the government debt exactly equal the long-run increase in taxation:

$$\dot{B} = rB + G - Z - \theta M = -(rB + G - \xi_0 - \theta M)/(\xi_1 - 1) \quad (B(0) = B_0). \tag{36}$$

Without a suitable tax rate ($\xi_1 > 1$), the government debt explodes and eventually the deficit has to be financed by money or taxes (cf., Sargent and Wallace, 1981).

IV.1. *Monetary Policy*

We are interested in the long-run effects of open-market operations that increase the monetary growth rate. For the case of weakly separable preferences, it can be shown that the relevant multipliers are:

$$\left.\frac{dC(\infty)}{d\theta}\right|^{BF} = \left.\frac{dK(\infty)}{d\theta}\right|^{BF} = 0, \tag{37}$$

$$\left.\frac{dM(\infty)}{d\theta}\right|^{BF} = \{\gamma r U F_{KK} + \beta(\alpha + \beta)[\gamma(M+B)F_{KK} - (rw/U)]\} M/\det(J''') < 0, \tag{38}$$

where

$$\det(J''') = -\{r(r+\theta)\Delta + \beta(\alpha+\beta)[\gamma(r+\theta)BF_{KK} + \gamma\theta M F_{KK} - (1-\gamma)\theta r]\} > 0.$$

Obviously, with infinite lives, bond-financed monetary growth is also superneutral and does not affect the real interest rate, capital or consumption. The effects on real money balances and seignorage revenues are exactly the same for $\beta = 0$ as with tax-financed monetary growth (see Section II.1). Since seignorage revenues increase, the government can afford to service a larger stock of government debt. There are, as in Section II.1, no effects of monetary growth on total real human and non-human wealth. This reflects the Ricardian debt equivalence proposition (e.g. Barro, 1974), because the increase in human wealth arising from the reduction in lump-sum taxes under the tax-financed increase in monetary growth is exactly the same as the increase in bonds under the bond-financed increase in monetary growth:

$$\left.\frac{dB(\infty)}{d\theta}\right|_{\beta=0}^{BF} = \left.\frac{dH(\infty)}{d\theta}\right|_{\beta=0}^{TF} = -\left.\frac{dM(\infty)}{d\theta}\right|_{\beta=0}^{BF,\,TF} = \frac{M}{r+\theta} > 0. \tag{39}$$

Also, the long-run effects on social welfare are independent of the residual mode of finance.

More interesting is the fact that, under weakly separable preferences, bond-financed monetary growth is superneutral, irrespective of the length of private agents' planning horizons.[8] Obviously, for the case of weakly separable preferences, bond-financed increases in monetary growth unambiguously worsen social welfare, even when lives are finite. However, when lives are finite *and* preferences are non-separable in consumption and real money balances, monetary growth can have real effects in the long run. This may be seen from the steady-state relationships:

$$F(K,1) = \Phi[F_K(K,1)-\delta+\theta]\{[F_K(K,1)-\delta]A+F_L(K,1)-Z\}+\delta K+G, \tag{40}$$

$$[F_K(K,1)-\delta-\alpha]\{[F_K(K,1)-\delta]A+F_L(K,1)-Z\} = \beta(\alpha+\beta)A. \tag{41}$$

With separable preferences ($\Phi' = 0$) non-neutralities only appear in the short run, but with finite lives and non-separable preferences real variables are affected by changes in the rate of monetary growth both during the transient path towards long-run equilibrium and in the steady state. The reason is that monetary growth affects the nominal rate of interest, which in turn affects the proportion spent on consumption of physical goods. It can be shown that a bond-financed increase in monetary growth decreases (increases) capital and consumption when the income (substitution) effect dominates, that is, when $\Phi' < 0\ (> 0)$. For a CES sub-utility function, this occurs when the elasticity of substitution between goods and real money balances, σ, is less (greater) than unity. When the substitution effect is strong enough, social welfare may be improved by increases in monetary growth, since the effect of the increase in consumption on social welfare could outweigh the effect of the fall in real money balances.

IV.2. *Fiscal Policy*

The effect of a bond-financed fiscal expansion is to reduce the steady-state level of the real interest rate and increase the capital stock:

$$\left.\frac{dK(\infty)}{dG}\right|^{BF} = \beta(\alpha+\beta)\,w(r+\theta)/[U\det(J''')] \gtrless 0. \tag{42}$$

[8] If the tax rule (35) also depends positively on the stock of outstanding debt and lives are finite, it is possible for monetary growth to affect real variables, even if preferences are weakly separable, ($\Phi' = 0$). The increase in monetary growth raises seignorage revenues and consequently permits the servicing of a greater stock of bonds. This increases lump-sum taxes and cuts human wealth. The net effect on consumption and the capital stock is, however, ambiguous.

Again, a fiscal expansion only affects the capital stock when lives are finite. It can easily be shown that for small rates of monetary growth a bond-financed fiscal expansion is less (more) expansionary than a money-financed fiscal expansion when the government (private sector) has a debt with the private sector (government). Hence, for $B > 0$

$$\left.\frac{dK(\infty)}{dG}\right|^{TF} < 0 < \left.\frac{dK(\infty)}{dG}\right|^{BF}_{\theta=0} < \left.\frac{dK(\infty)}{dG}\right|^{MF}_{\theta=0} \tag{43}$$

must hold and therefore a bond-financed fiscal expansion leads to more crowding out of private consumption than a money-financed fiscal expansion.

It can be shown that for weakly separable preferences the long-run effects of a bond financed fiscal expansion on the steady-state levels of real money balances and bonds are (evaluated at small rates of monetary growth):

$$\left.\frac{dM(\infty)}{dG}\right|^{BF}_{\theta=0} = \frac{F_{KK}}{\det(J''')}\{r(1-\gamma)U + \beta(\alpha+\beta)[(1-\gamma)(A+B) - \gamma M]\}, \tag{44}$$

$$\left.\frac{dB(\infty)}{dG}\right|^{BF}_{\theta=0} = -\frac{1}{r} + \frac{\beta(\alpha+\beta)\,wBF_{KK}}{U\det(J''')} < 0. \tag{45}$$

When seignorage revenues are small (as the expressions are evaluated around $\theta = 0$), an increase in government spending implies that there are less funds for servicing the government debt and therefore the government debt must fall in the long run (even though it increases in the short run). When lives are infinite, there are no effects on capital, human wealth or the rate of interest. Total wealth falls, because both real money balances and bonds fall. The fall in bonds is exactly the same as the fall in human wealth in the tax finance case, which reflects once again the Ricardian debt equivalence proposition. When lives are finite, the effect on real money balances is ambiguous. The fall in total wealth depresses real money balances, but the lower rate of interest increases them. Note that total wealth falls despite the fact that equity increases and that human wealth increases.

V. CONCLUDING REMARKS

A summary of the results for the case that preferences are weakly separable in the consumption of goods and real money balances is presented in Table 1. The validity of well-established classical policy prescriptions does not survive the explicit consideration of finite lives. Monetary growth is superneutral in the standard Sidrauski-type analysis based on infinite lives. However, with the Yaari Blanchard approach of finite lives, an increase in the monetary growth increases inflation one-for-one, cuts the real interest rate and raises capital, consumption, human wealth (via associated cuts in lump-sum taxes, higher wage income, and a lower real interest rate) and total wealth in the long run. The effect on the nominal interest rate and thus on real money balances is ambiguous. It follows that a tax-financed increase in monetary growth can, with finite lives, increase social welfare when the positive effect on consumption outweighs the possible negative effect on real money balances. On the other hand, superneutrality of monetary growth prevails when the residual mode of finance is bond issues and preferences are weakly separable. In this case, an increase in monetary growth is highly undesirable from a welfare point of view, as it results in a higher nominal interest rate and lower real money balances without improvements in capital or consumption, even when lives are finite. However, when lives are finite and preferences are non-separable, a bond-financed increase in monetary growth can have real effects. It increases capital and consumption when the substitution effect dominates the income effect, that is when the elasticity of substitution between goods and real money balances exceeds unity. When the substitution effect is strong enough, social welfare could increase.

With infinite lives a fiscal expansion has no real effects and leads to 100% crowding out of private consumption, irrespective of the residual mode of finance. The Ricardian debt equivalence result plays a role in this case, because the fall in human wealth in the tax-finance case is exactly the same as the additional fall in real money balances in the money-finance case and the fall in real bonds in the bond-finance case. With finite lives there are real effects of a fiscal expansion. A tax-financed fiscal expansion leads to an increase in the real interest rate, a fall in the capital stock and more than 100% crowding out of private consumption. Both a bond-financed and a money-financed fiscal expansion lead to a lower real interest rate, higher capital stock and less than 100% crowding out of private consumption. A bond-financed fiscal expansion is less expansionary and leads to more crowding out than a money-financed fiscal expansion. The reason that, with finite lives, a tax-financed fiscal expansion is worse than a bond-financed fiscal expansion is that total wealth falls by less in the latter case, as agents may not be alive to pay the taxes required to redeem the government debt eventually. It is no surprise that a fiscal expansion worsens social welfare, even though the money-finance and bond-finance modes of residual finance raise the capital stock, because government spending has been assumed to be of the 'hole-in-the-ground' variety. A meaningful welfare evaluation requires the introduction of government spending into the direct utility function. When government spending enters in a separable fashion, the results on comparative statics will be unaffected. In that case a fiscal expansion may increase social welfare when the weight on government spending is high enough.

Obvious directions for future research are to extend the model to allow for distortionary taxes, elastic labour supply, government spending in the utility function, money in the production function, more realistic assumptions about the birth and death processes and wage rigidities and unemployment. The next step would be a normative analysis of the optimal provision of public goods and optimal monetary policy.

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